Exam

Radiation Physics 2018-2019

Friday 9-11-2018: 9:00-12:00

NB5114.004

Read these instructions carefully before making the exam!

- **Write your name and student number on *every* sheet.**
- ***Make sure to write readable for other people than yourself. Points will NOT be given for answers in illegible writing*.**
- ***Language*; your answers have to be in English.**
- **Use a *separate* sheet for each problem.**
- **Use of a (graphing) calculator is allowed.**
- **This exam consists of 4 problems.**
- **The weight of the problems is Problem 1 (P1=23 pts); Problem 2 (P2=22 pts); Problem 3 (P3=22 pts): Problem 4 (P4= 23 pts). Weights of the various subproblems are indicated at the beginning of each problem.**
- **The grade of the exam is calculated as (P1+P2+P3+P4+10)/10.**
- **For all problems you have to write down your arguments and the intermediate steps in your calculation, *else the answer will be considered as incomplete and points will be deducted.***

PROBLEM 1 *Score: a+b+c+d+e=5+4+5+5+4=23*

The range R of a charged particle with kinetic energy E_{kin} in a certain material m is defined by:

$$R(E_{kin}) = \int_0^{E_{kin}} \left(-\frac{1}{S_m(E)}\right) dE$$

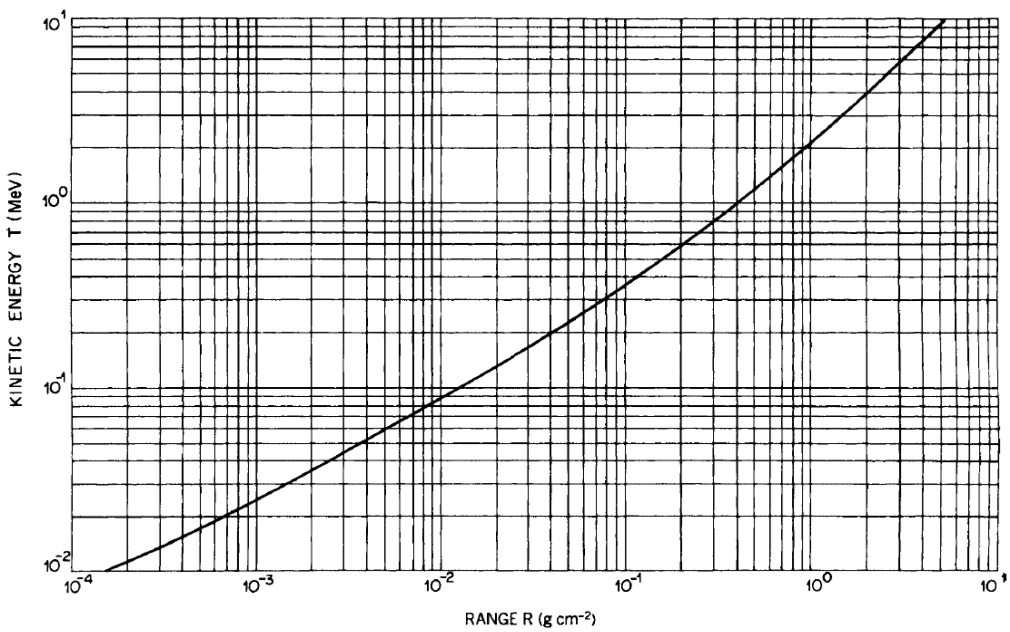
with S_m the stopping power of the material m.

a) Explain why this definition gives a good estimate of the depth to which a heavy charged particle penetrates a material but gives a poor indication of the depth to which a beta particle goes into a material? Does the definition result in an over- or underestimate of the range of beta particles? Explain.

A positron emerges perpendicularly from a 7 mm thick polycarbonate slab ($C_{16}H_{18}O_5$, density 1.20 g cm^{-3}). The energy of the emerging positron is 1.2 MeV.

b) Estimate the range (in cm) of a 1.2 MeV positron in polycarbonate.
c) Calculate the energy of the positron when it entered the slab.
d) Suppose a 1 MBq source of ^{11}C (β^+(100%): E_{max} = 0.960 MeV; $T_{1/2}$: 20.4 min)) is stored in the centre of a hollow spherical container. This container is made of polycarbonate and has an inner radius of 4 cm and a wall thickness equal to the range of most energetic positrons emitted by the source. Estimate the γ-flux (in MeV cm^{-2}s^{-1}) exactly at the outer surface of the container. You may neglect attenuation of the gamma flux by the container wall.
e) What material would you recommend for additional shielding?

Range (in g cm^{-2}) of beta particles in low Z materials



PROBLEM 2 *Score: a+b+c+d+e =6+4+4+4+4=22*

A narrow beam of 10^5 photons per second is perpendicularly incident on a slab of iron (Z: 26; A: 55.8; density: 7.9 g cm^{-3}). The iron slab is 4 mm thick. The beam consists for 40% out of 100 keV photons and for 60% out of 1.5 MeV photons.

a) For each of the two photon energies, mention and shortly describe (no formulas are needed) the main processes in which they interact and transfer energy to the iron slab.
b) Calculate the number of photons per second of each energy that are transmitted through the slab without interaction.
c) Calculate the linear attenuation coefficient for 1.5 MeV photons in iron using the atomic cross sections given in the table below. Check if your answer is consistent with the data from the other table.
d) How much energy (in MeV) is removed from the narrow beam per second by the slab?
e) How much energy is absorbed (in MeV) by the slab per second? Is this anwer consistent with your anwer in d)?

Mass attenuation and mass energy absorption coefficients (both in cm^2 g^{-1}) for photons in iron.

Photon energy (MeV)	$\frac{\mu}{\rho}$	$\frac{\mu_{en}}{\rho}$
0.1	0.37	0.22
1.5	0.049	0.024

Atomic cross sections of iron (in barn per atom)

Interaction	σ at 100 keV	σ at 1.5 MeV
Photoelectric	19.0	0.015
Compton	12.0	4.5
Pair production	0.0	0.035

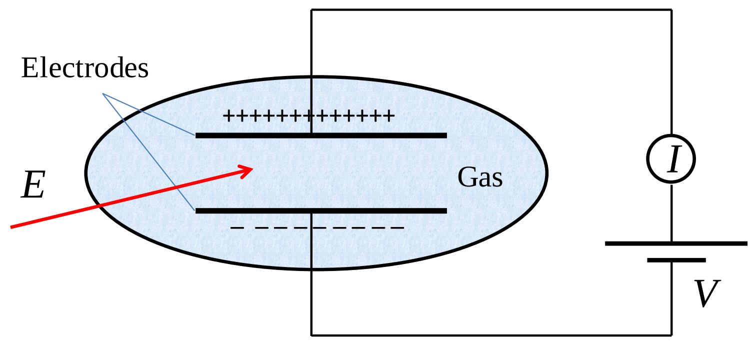
PROBLEM 3 *Score: a+b+c+d+e=5+5+4+4+4=22*

A sample containing ^{40}K (half life: $1.25 \cdot 10^9$ year) is placed on a scintillator detector. ^{40}K emits a gamma ray of 1460 keV with a probability of 11% per decay. The efficiency for detecting this gamma ray is 15%. A 1 hour background measurement (thus without the sample) results in 1800 counts. The sample itself is measured for 10 minutes resulting in 600 gross counts.

a) Calculate the net count rate and the activity of the sample.
b) Calculate the standard deviation in the net count rate and in the activity of the sample.
c) Without doing additional background counting, how long would the sample have to be counted in order to obtain a standard deviation of 5% in the net count rate.
d) Estimate the probability that during the first second of the background measurement 4 counts are measured.
e) During a 2 minute measurement of the background an experimenter measures 40 counts in the first minute and 20 counts in the second minute. Estimate the probability that this happens.

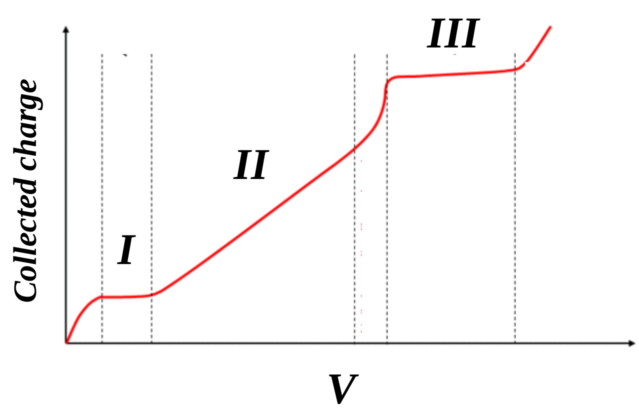
PROBLEM 4 *Score: a+b+c+d+e =6+4+4+4+5=23*

Ionising radiation can be detected with a gas-filled ion chamber in which a cathode and anode are mounted (see figure below).



Scheme of a gas-filled ion chamber.

As a function of the voltage between the electrodes we can discriminate different regions of operation of the ion chamber. Three of these regions (I, II en III) are indicated in the figure below in which the charge collected on the electrodes (log-scale) is plotted as a function of the applied voltage difference (linear scale).



Collected charge (log scale) as a function of applied voltage V (linear scale) for a gas-filled ion chamber.

a) Explain the mechanisms of the ion collection in region I, II and III.
b) The W-values of a certain gas-filled ion chamber for electrons, alpha's and carbon ions are 25, 40 and 50 eV per ion pair, respectively. Calculate the width (FWHM in keV) of the total energy peaks of each of these particles if they hit the detector with a kinetic energy of 1 MeV.

Problem continues on next page

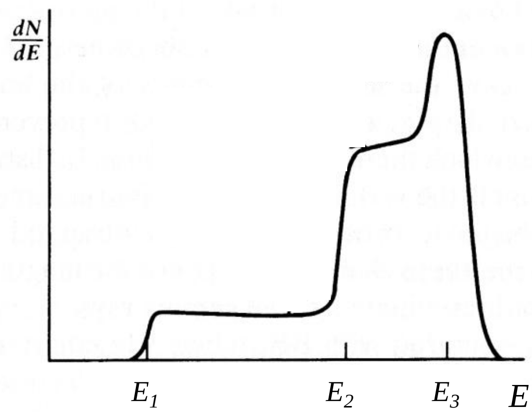
c) A stream of 3 MeV particles is hitting the ion chamber with a rate of 10^6 particles per second. The saturation current (region I) measured in this situation is 9.6 nA. Figure out if these particles are electrons, alpha's or carbon ions.

An ion chamber filled with ^{3}He gas can be used for the detection of slow neutrons. The neutron detection is based on the reaction,

$$^3_2\mathrm{He} + \mathrm{n} \rightarrow {}^3_1\mathrm{H} + p$$

d) Calculate the Q-value of this reaction.

A measurement with such an ion chamber filled with ^{3}He gas resulted in the spectrum presented in the figure below.



e) Explain the shape of the spectrum and calculate the values of the energies that are indicated on the horizontal axis.

Mass differences Δ (in MeV) of some particles and isotopes

Isotope	Mass difference
n	8.0714
$^1_1$H	7.2890
$^3_1$H	14.9500
$^3_2$He	14.9313

Solutions

PROBLEM 1

a)

Due to the small mass of the electron, heavy charge particles in collisions with electrons can only transfer a small fraction of their kinetic energy and are deflected only over small angles. Due to this they essentially follow a straight path thus the small steps in distance between collisions line up and the integral will approximate the penetration depth in the material. In contrast, beta particles can lose up to all their energy in a single collision and undergo large deflections resulting in a tortuous path and the integral (which assumes a straight path) will lead to an overestimate of the range of the beta particle.

b)

Polycarbonate is a low Z material thus we can use the graph that is supplied. From the graph we estimate that the range for a positron of 1.2 MeV is approximately 0.5 g cm^{-2}. The range in centimetres is $0.5/1.2 = 0.42$ cm.

c)

The positron already travelled 0.7 cm through polycarbonate so its total range is: $0.42 + 0.7 = 1.12$ cm. Converting this to $1.12 \times 1.2 = 1.34$ g cm^{-2}, and using the graph again we find that the initial kinetic energy is approximately 3 MeV.

d)

The source emits 1 positron for every decay. These positrons are all stopped in the wall of polycarbonate as this is equal to the range of the most energetic positrons. This range is (again use the graph) 0.4 g cm^{-2} or $0.4/1.2 = 0.3$ cm. Consequently, the outer radius of the container is $4 + 0.3 = 4.3$ cm. After coming to rest the positrons will annihilate with one of the electrons in the polycarbonate and two gamma's of 511 keV are emitted. As all these gamma escape from the container the flux at the surface is:

$$\frac{1 \cdot 10^6\ (\mathrm{Bq}) \times 0.511(\mathrm{MeV}) \times 2(2\ \gamma\ \mathrm{per\ Bq})}{4\pi\ (4.3)^2} = 4400\ \mathrm{MeVcm^{-2}s^{-1}}$$

e)

Some additional high density, high Z material around the sphere would be most effective to shield for the 0.511 MeV gamma rays. Lead would be an option.

PROBLEM 2

There are three main processes by which photons can interact with matter.
Photoelectric effect. In the photoelectric effect all energy of the photon is transferred to an (mainly inner shell) electron. This electron takes away the photon energy minus the electron binding energy as kinetic energy which is then transferred to the material by successive collisions with other electrons.
Compton scattering. The photon scatters on a loosely bound outer shell electron and transfers part of its energy to this electron. The electron transfers this energy by collisions with electrons to the material. The remaining energy is taken away by the scattered photon that may interact further with or escape from the material.
Pair production. Pair production is only possible for photons that have energies higher than two times the rest mass energy (2 times 511 keV) of the electron. In that case the photon can produce an electron positron pair that takes away the energy above 1022 keV as kinetic energy. This energy is transferred to the material by collisions with electrons to the material. The positron will annihilate with an electron and two 0.511 keV photons are emitted back to back. These photons may interact further with or escape from the material.

For the low energy photon only the photoelectric effect and Compton scattering are possible. From the table with the atomic cross sections we conclude that both effects are more or less equally important.

For the high energy photon all three processes are possible, but the Compton effect is the most important process.

b)
The fraction of photons that are transmitted without interaction is given by: $e^{-\mu d}$ with μ the linear attenuation coefficient and d the thickness of the slab.

For the 100 keV photons we have: $\mu = \left(\frac{\mu}{\rho}\right)\rho = 0.37 \times 7.9 = 2.9\ \mathrm{cm}^{-1}$ and $e^{-\mu d} = e^{-2.9\times 0.4} = 0.31$

For the 1.5 MeV photons we have: $\mu = \left(\frac{\mu}{\rho}\right)\rho = 0.049 \times 7.9 = 0.39\ \mathrm{cm}^{-1}$ and $e^{-\mu d} = e^{-0.39\times 0.4} = 0.86$

Thus, the numbers of photons transmitted per second without interaction are:

For the 100 keV photons: $0.4 \times 10^5 \times 0.31 = 12400$ photons per second.
For the 1.5 MeV photons: $0.6 \times 10^5 \times 0.86 = 51600$ photons per second.

c)
The total atomic cross section σ for 1.5 MeV photons is $\sigma = 0.015 + 4.5 + 0.035 = 4.55$ barn. The linear attenuation coefficient is given by $\mu = N_{Fe}\sigma$ with N_{Fe} the number

of iron atoms per cm^2. $N_{Fe} = \frac{\rho_{Fe}}{55.8} N_a = \frac{7.9}{55.8} \times 6 \cdot 10^{23} = 8.5 \cdot 10^{22}$ and $\mu = 8.5 \cdot 10^{22} \times 4.55 \cdot 10^{-24} = 0.39\ \mathrm{cm}^{-1}$.

Yes, consistent: $\frac{\mu}{\rho} = \frac{0.39}{7.9} = 0.049\ \mathrm{cm}^2\ \mathrm{g}^{-1}$.

d)
Energy flux of beam into the slab is: $0.4 \times 10^5 \times 0.1 + 0.6 \times 10^5 \times 1.5 = 94000\ \mathrm{MeVs}^{-1}$.
Energy flux of beam out of the slab is (this is carried by the unscattered photons, these numbers were calculated in b)): $12400 \times 0.1 + 51600 \times 1.5 = 78640\ \mathrm{MeV\ s}^{-1}$.

The energy (in MeV) removed from the narrow beam per second by the slab is then given by the difference of the flux in and flux out: $94000\ \mathrm{MeVs}^{-1} - 78640\ \mathrm{MeV\ s}^{-1} = 15360\ \mathrm{MeV\ s}^{-1}$.

e)
We first calculate the energy transmitted by the slab. For this we need the linear mass energy absorption coefficients to calculate the fraction of energy transmitted.

For the 100 keV photons we have: $\mu = \left(\frac{\mu}{\rho}\right)\rho = 0.22 \times 7.9 = 1.7\ \mathrm{cm}^{-1}$ and $e^{-\mu d} = e^{-1.7 \times 0.4} = 0.50$

For the 1.5 MeV photons we have: $\mu = \left(\frac{\mu}{\rho}\right)\rho = 0.024 \times 7.9 = 0.19\ \mathrm{cm}^{-1}$ and $e^{-\mu d} = e^{-0.19 \times 0.4} = 0.93$

Total energy transmitted is: $0.4 \times 10^5 \times 0.1 \times 0.50 + 0.6 \times 10^5 \times 1.5 \times 0.93 = 85700\ \mathrm{MeVs}^{-1}$.

Total energy absorbed in the slab: $94000\ \mathrm{MeVs}^{-1} - 85700\ \mathrm{MeV\ s}^{-1} = 8300\ \mathrm{MeV\ s}^{-1}$

The energy absorbed in the slab is less than the energy removed from the narrow beam (see d)). This is consistent as also part of the scattered photons (so removed from the beam) are still transmitted through the slab but at lower energy and in general in different directions than the beam direction.

PROBLEM 3

a)

Background count rate is: $r_b = \frac{1800}{60} = 30$ cpm. Gross count rate is: $r_g = \frac{600}{10} = 60$ cpm. Net count rate is: $r_n = r_g - r_b = 60 - 30 = 30$ cpm. We only count 15% of all emitted gammas (efficiency) and the gamma is only emitted in 11% of the decays. Thus the activity is given by $A = \frac{r_n}{0.15 \times 0.11 \times 60} = 30.3$ Bq (remember to convert the count rate to cps).

b)

Standard deviation in background count rate is $\sigma_b = \frac{\sqrt{1800}}{60} = 0.71$ cpm. Standard deviation in gross count rate is $\sigma_g = \frac{\sqrt{600}}{10} = 2.4$ cpm. The standard deviation in the net count rate is $\sigma_n = \sqrt{\sigma_b^2 + \sigma_g^2} = \sqrt{(0.71\,)^2 + (2.4)^2} = 2.5$ cpm. The standard deviation in the activity is $\sigma_A = \frac{\sigma_n}{0.15 \times 0.11 \times 60} = \frac{2.5}{0.15 \times 0.11 \times 60} = 2.5$ Bq.

c)

The standard deviation in the net count rate is:

$$\sigma_n = \sqrt{\sigma_b^2 + \sigma_g^2} = \sqrt{\sigma_b^2 + \left(\frac{\sqrt{N_g}}{T_g}\right)^2} = \sqrt{\sigma_b^2 + \frac{N_g}{T_g^2}} = \sqrt{\sigma_b^2 + \frac{r_g}{T_g}}$$

with N_g and T_g the number of gross counts and the time of gross counting, respectively. We need to find a T_g such that:

$$\frac{\sqrt{\sigma_b^2 + \frac{r_g}{T_g}}}{r_n} = 0.05$$

is satisfied.

Solving, we find:

$$\sqrt{\sigma_b^2 + \frac{r_g}{T_g}} = 0.05 r_n \Rightarrow \sigma_b^2 + \frac{r_g}{T_g} = (0.05 r_n)^2 \Rightarrow \frac{r_g}{T_g} = (0.05 r_n)^2 - \sigma_b^2 \Rightarrow$$

$$T_g = \frac{r_g}{(0.05 r_n)^2 - \sigma_b^2} = \frac{60}{(0.05 \times 30)^2 - (0.71)^2} = 34.4 \text{ min}$$

d)

During the first second $1 \times \frac{30}{60} = 0.5$ counts are expected. We can use Poisson statistics because it is a very long lived radionuclide. This means that the probability on 4 counts is:

$$P_4 = \frac{\mu^4 e^{-\mu}}{4!} = \frac{0.5^4 e^{-0.5}}{4!} = 0.0016$$

e)

In a one minute interval we expect 30 counts. The probability to observe 40 counts in the first one minute and 20 counts in the second one minute interval is the product of the probabilities for each interval because radioactive decay does not have a memory and due to this the two probabilities are independent:

$$P_{40} \times P_{20} = \frac{\mu^{40} e^{-\mu}}{40!} \times \frac{\mu^{20} e^{-\mu}}{20!} = \frac{30^{40} e^{-30}}{40!} \times \frac{30^{20} e^{-30}}{20!} = 0.0139 \times 0.0134$$

$$= 1.86 \cdot 10^{-4}$$

PROBLEM 4

a)

Region I: Radiation will ionize gas molecules and the electric field will separate the ion pairs (electron and ionized gas molecule). The charge will be collected on the electrodes. In the region before region I the electric field is not strong enough and recombination of ion pairs occurs. In region I the field is strong enough to collect all charge and a plateau region is reached. This is the operational region of the 'ion chamber'.

Region II: This is the region of the proportional counter that is based on the principle of gas multiplication. The applied voltage is high enough that the electric field can accelerate the electrons from ionization to such energies (higher than the ionization energy of the gas molecules) that these electrons become ionizing themselves. This liberates extra electrons and the amount of charge collected per event is larger. The extra amount of charge (electrons) is proportional to the applied voltage. The amount of collected charge is proportional to the energy of the radiation.

Region III: This is the Geiger Mueller region. Space charge of the positive ions prohibits that the electric field further increases. Every event (radiation particle) results in the same amount of charge. The collected charge is independent of the energy of the radiation.

b)

The expected number N of ion pairs is 1 MeV/W. This results in 50000, 25000 and 20000 ion pairs for electrons, alpha's and carbon ions, respectively.

Assuming Poisson statistics we have for the relative resolution R at 1 MeV,

$$R = \frac{FWHM}{E} = \frac{2.35\sigma}{E} = \frac{2.35k\sqrt{N}}{kN} = \frac{2.35}{\sqrt{N}}$$

Thus, R is 0.01; 0.015 and 0.017 for electrons, alpha's and carbon ions, respectively.

Thus the FWHM at 1 MeV is $R \times E = 10; 15;$ and 17 keV for electrons, alpha's and carbon ions, respectively.

c)

A current of 9.6 nA corresponds to $9.6 \cdot 10^{-9}$ C per second or $\frac{9.6 \cdot 10^{-9}}{1.6 \cdot 10^{-19}} = 6 \cdot 10^{10}$ electrons per second. This is equal to the total amount of electrons produced by the stream of particles:

$$\frac{3 \cdot 10^{6} \times 1 \cdot 10^{6}}{W} = 6 \cdot 10^{10} \Rightarrow W = 50 \text{ eV per ion pair}$$

The particles are carbon ions.

d)

The Q-value is given by:

$$\Delta({}^{3}_{2}\mathrm{He}) + \Delta(\mathrm{n}) - \Delta({}^{3}_{1}\mathrm{H}) - \Delta({}^{1}_{1}\mathrm{H}) = 14.9313 + 8.0714 - 14.9500 - 7.2890$$
$$= 0.7637 \text{ MeV}$$

e)
The peak at E_3 is a 'full energy' peak, both the energy of the triton and the proton are completely absorbed in the gas of the counter. Total absorbed energy is equal to the Q-value so E_3=0.764 MeV.
The energy distribution between the triton and the proton follows from conservation of energy and momentum.

$Q = E_t + E_p = \frac{1}{2}m_t {v_t}^2 + \frac{1}{2}m_p {v_p}^2$ en $m_t v_t + m_p v_p = 0$

Which can be solved $E_t = \frac{Q}{1+\frac{m_t}{m_p}} = \frac{Q}{4} = 0.191$ MeV and $E_p = 0.573$ MeV.

In the plateau directly left of the 'full energy' peak all energy of the proton and part of the energy of the triton is absorbed in the gas of the counter. The remaining energy of the triton is deposited in the wall of the counter. The steep transition occurs at E_2=0.573 MeV, when the complete energy of the triton is deposited in the wall. The second plateau starts at the situation when only the energy of the triton is absorbed, (E_1=0.191 MeV) and in the rest of the plateau also part of the proton energy is absorbed.